Hydropower Plants: Generating and Pumping Units
Solved Problems: Series 3

1 STUDY ABOUT A HYDRAULIC POWER PLANT

1.1 Basic calculation for a specific speed

Here, the procedure for choosing the type of turbine for a given power plant and the fundamentals of Pelton turbines are studied. In this exercise, the hydraulic power plant of interest has a rated discharge Q = 28 m^3 s^{-1} and an available head H = 350 m. For calculations, use the following values for gravity acceleration and water density:

$$g = 9.81 \text{ m s}^{-2},\ \rho = 1'000 \text{ kg m}^{-3}$$

1) For the generating unit, it is planned to install a generator featuring 50 poles. Deduce the rotational frequency n of the turbine. For the grid frequency f_{grid}, use the value of f_{grid} = 60 Hz.

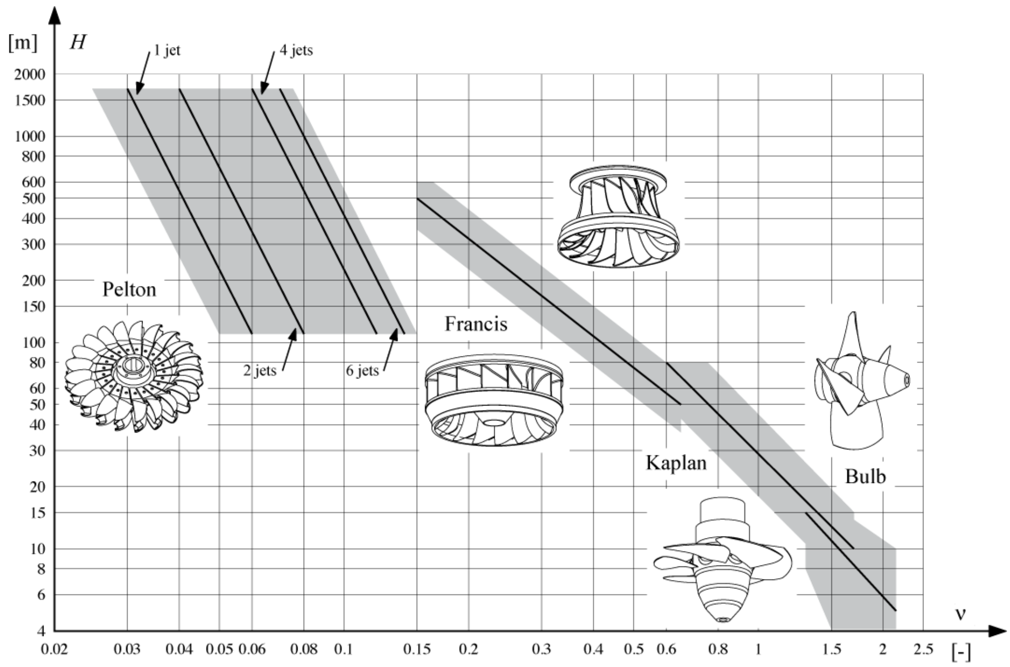

Figure 1: Turbine type depending on specific speed ν and head H

2) Calculate the specific speed ν and, using Figure 1, select a suitable turbine type for the power plant.

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1.2 Study about a Pelton turbine

Pelton turbines are mostly used for high head hydropower plants. Their geometry is based on water splitting buckets (see Figure 2). Answer the following questions using the values provided in Section 1.1.

3) In which head range are Pelton turbines used?
4) What is the difference between horizontal and vertical axis Pelton turbines? For a large-sized Pelton runner, explain which one is the most appropriate and why.
5) Explain the difference between Pelton and Francis turbines using the following keywords: *impulse turbine, reaction turbine, velocity, pressure*

For Pelton turbines, an output power P can be calculated by the following equation based on the velocity triangle in Figure 2, when the bucket and mechanical losses are negligible:

$$P = \rho Q \left\{ \frac{\left(\vec{W}_1 + \vec{U}_1\right)^2}{2} - \frac{\left(\vec{W}_{\bar{1}} + \vec{U}_{\bar{1}}\right)^2}{2} \right\} = \rho Q \left(\vec{W}_1 \cdot \vec{U}_1 - \vec{W}_{\bar{1}} \cdot \vec{U}_{\bar{1}} \right) \quad (1)$$

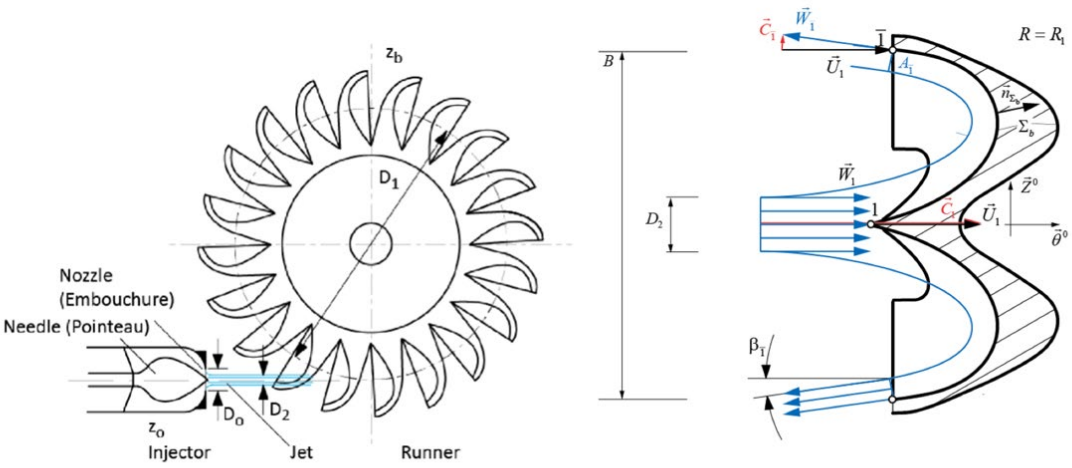

Figure 2: Pelton turbine bucket and velocity triangle ($\vec{C}$ the absolute discharge velocity, $\vec{U}$ the bucket tangential velocity, $\vec{W}$ the relative discharge velocity)

6) Deduce an expression equivalent to $\vec{W}_1 \cdot \vec{U}_1 - \vec{W}_{\bar{1}} \cdot \vec{U}_{\bar{1}}$ and express it as a function of C_1, U_1 and $\beta_{\bar{1}}$ based on the vectorial relationship illustrated in Figure 2. Here, the amplitude of the relative velocity $\left|\vec{W}_{\bar{1}}\right|$ can be assumed the same as $\left|\vec{W}_1\right|$.
7) The jet velocity is $C_1 = \varphi_s \sqrt{2gH}$ with φ_s being a velocity coefficient, taking nozzle and volute (if present) losses into account. Assuming $\varphi_s = 0.97$, calculate the nozzle efficiency. Hint: Nozzle efficiency may be formulated as $\eta_s = \dfrac{0.5C_1^2}{gH}$.
8) From Eq. (1) and the expression you computed in question 6), deduce the output power P as a function of the velocity ratio $\dfrac{U_1}{C_1}$. Then, plot the curve of the output power P as a function

of the velocity ratio, and check the optimal bucket velocity $U_{1,opt}$. What do the points $\frac{U_1}{C_1}=0$ and $\frac{U_1}{C_1}=1$ represent?

9) Calculate the output power P for the designed power plant, at the velocity ratio $\frac{U_1}{C_1}=0.6$ with the jet diameter $D_2 = 0.4709$ m and the outlet flow angle $\beta_{\bar{1}} = 5°$. Furthermore, calculate the maximum output power P_{max}.

10) What is the mechanical risk for the generator when connecting the turbine to the grid at runaway?

11) Express the optimum diameter of the Pelton runner (i.e. consistent with the maximum output power condition) as a function of the available head H and angular rotational frequency ω.

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2 APPLICATION OF EULER EQUATION TO PELTON TURBINE

The Euler equation can be applied to the calculation of the transformed specific energy E_t for Pelton turbines. For these turbines, the transformed specific energy E_t can be calculated in the following way:

$$E_t = \vec{C}_1 \cdot \vec{U}_1 - \vec{C}_{\bar{1}} \cdot \vec{U}_{\bar{1}}$$

In Figure 3, a sketch of the flow in a Pelton bucket is shown, as well as the corresponding velocity triangles (Q: discharge, C: absolute jet velocity, U: absolute runner rotational velocity, W: relative velocity).

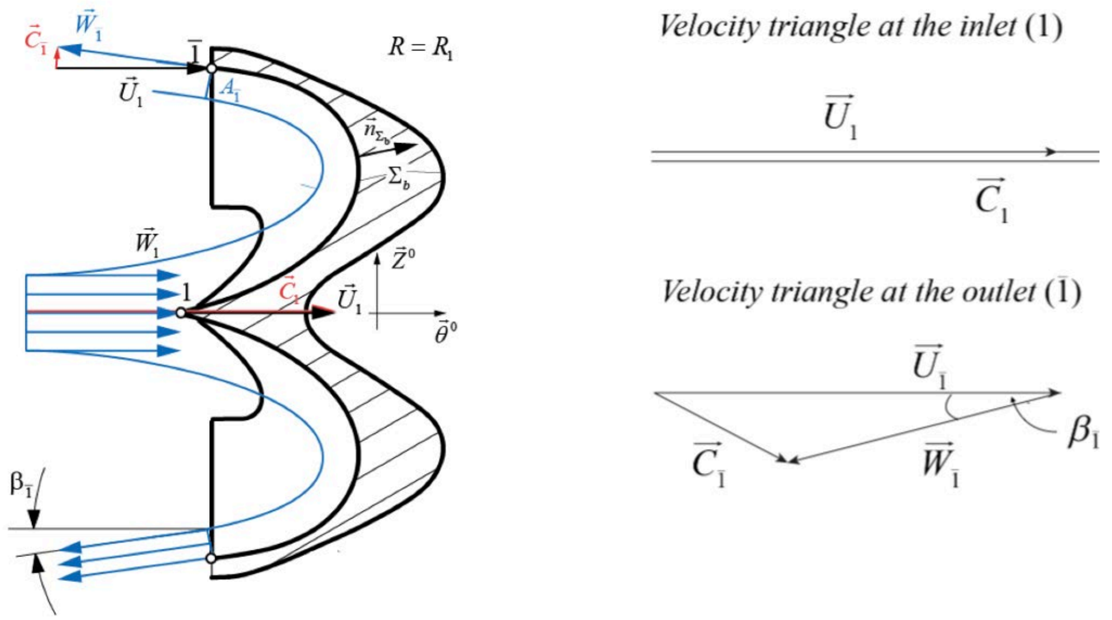

Figure 3: Sketch of the flow in a Pelton bucket, with the corresponding velocity triangles at the inlet and the outlet.

12) Using the Euler equation and the velocity triangles at the inlet and the outlet, express the transformed specific energy E_t as a function of the relative velocity $W_{\bar{1}}$, the absolute jet velocity C_1, the rotational velocity of the runner U and the angle $\beta_{\bar{1}} = \beta$.
13) Assuming that $\left|\overrightarrow{W_1}\right| = \left|\overrightarrow{W_{\bar{1}}}\right|$, give the transformed specific energy E_t as a function of C_1, U and β.
14) Express the transferred power P_t as a function of ρ, Q, C_1, U and β. You can consider $Q_t = Q$.
15) If your calculations are correct, you should end up with the same expression of the power as in Question 8). However, here we considered the transferred power P_t, whereas in Q.8) we referred to the output power *P*. Which assumption made in Q.5) causes these two power expressions to be the same?
On the other hand, which assumption from Q.5) doesn't have an impact on the difference that actually exists between the transferred power P_t and the output power *P*?
16) For a given discharge Q and absolute jet velocity C_1, sketch the evolution of P_t as a function of U.
17) Express the maximum transformed power $P_{t_{max}}$ and the optimal runner velocity U_{opt} at the maximum power as a function of ρ, Q, C_1 and β.